A Non Destructive Technique for the Characterization of Soil Constitutive Parameters using a Monopole Probe

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Abstract - A microwave non-destructive experimental tool based on a monopole antenna is used for the determination of the complex permittivity of soils in a broad frequency band. The monopole mounted on a ground plane is buried in the soil, and its reflection coefficient measured at the feed point by a vector network analyzer (VNA) depends on the dielectric properties of the surrounding medium at a given frequency. In particular, our study is focused on the evaluation of the change of dielectric properties of different soils with the moisture content. After a detailed parametric study to define the best antenna geometry parameters adapted to the soils to be considered, we have developed original algorithms for short and long monopoles based on 2 main steps. The first step consists in estimating the complex permittivity of the soil, a mean value or as a function of the frequency, with 3 specific algorithms: the resonant frequency algorithm, the Prony High Resolution (HR) approach and a fitting using a fractional function. In a second step, the volumetric moisture content of the soil is deduced from the real part of the complex permittivity using parametric relations based on the polynomial Topp's formula, and an extended version of the CRIM (Complex refractive Index Model) formulation. The validation of the data processing developed has been made on experimental data of complex reflection coefficients obtained in the frequency band [0.1;4] GHz associated with different types of soils (sands and silts).

Keywords – Complex permittivity, Reflection coefficient, Soil, Moisture, Monopole, Electromagnetic waves

I. INTRODUCTION

In ground-penetrating radar (GPR), non-destructive techniques methods appear particularly attractive as they allow to measure by reflection or transmission the effective complex permittivity of soils at different locations and depth [1]. Such techniques, as visualized on Fig. 1, are generally divided into two categories, surface and drill-hole based measurements, which use open-ended transmission line or free-space methods. Among surface reflection techniques, the coaxial-excited monopole probe with a circular ground plane appears as a particularly simple and efficient technique to characterize the dielectric properties of the near surface of soft soils in a broad frequency band [0.1;4] GHz. The probe, which is supposed to be immersed in a semi infinite dielectric soil, is made of a thin-wire element which is an extension of the inner conductor of the coaxial line. As the dimensions of the coaxial line are chosen so that a Transverse Electromagnetic (TEM) mode is the only propagating mode in the line, a TEM input impedance (Z11) can be defined at coaxial line interface. The wire element radiates an electromagnetic waves into the medium, and its complex effective permittivity can be estimated from the measurement of the reflection coefficient (S11) or the input impedance (Z11) using a Vector Network Analyzer (VNA). Thus, in a wide frequency band and in a given medium, the electrical length of the antenna can range from electrically small to electrically long: a short probe of one-tenth wavelength (or less) long is found suitable for localized measurements, and a probe of about a quarter wavelength is suggested for an average measurement over a larger sample volume. Specific modeling is required for each probe, and it can be expressed analytically when the circular ground plane of the monopole can be assumed infinite.

The study proposed in this paper focuses on the validation of the monopole probe technique for the estimation of the moisture content of soils. In general, a nonlinear inverse problem has to be solved which includes two fundamental steps: first, the effective complex permittivity of the material has to be estimated from the reflection coefficient of the probe at several frequencies. First, two types of modeling based on analytical (Wu (1961) [2-3], and Smith and Norgard (1985) [4]) and numerical approaches (based on the FDTD) have been used and compared. The inversion algorithms for the short and the long monopoles aims the estimation of the complex permittivity of the surrounding medium using the analytical modelling validated by numerical simulations. Afterwards, the use of a mixture law allows to solve a nonlinear inverse problem to obtain the parameters characterizing the constituents of the surrounding medium. In general, soil forms a heterogeneous material made of a mineral-air-water mixture. Thus, its dielectric properties are closely dependent on the dielectric constants of the individual constituents, the volume fractions of the components, the geometric characteristics of the constituents, and the electrochemical interactions between the constituents. Concerning the dependence of a medium with moisture, several
models have been proposed, each with its own characteristics and variables. Fundamental developments have been associated to the empirical formula of Topp et al (1980) [5], to a three phase model formulated by Polder and Van Santen (1946), and by de Loor (1983) [6], and to a four phase model proposed as a semi-disperse model by Wang and Schmugge (1980), a semi-empirical power-law model by Dobson et al. (1985), Ulaby et al (1986) [7], and Peplinski et al. (1995), and a generalized refractive dielectric model by Mironov et al. (2004) [8]. Recently, Boyarskii et al. (2002) [9] have suggested a model of the complex permittivity of bound water versus frequency in wet soils, and Jones et al. (2000) [10] have studied the effects of particle size on the bulk dielectric constant. In the present study, the volumetric moisture content of the soil has been deduced from the real part of the complex permittivity using parametric relations based on the polynomial Topp’s formula [5], and an extended version of the CRIM (Complex refractive Index Model) formulation [7,11].

The validation of the experimental setup has been performed on experimental measurements associated with different types of sands with several moisture contents and with silts with different compaction levels.

II. ANTENNA GEOMETRY

The monopole probe, shown on Fig. 2, is composed of a thin-wire element of finite length $h$ positioned on a circular disk of radius $d$. Both elements are assumed perfectly conductive. The choice of both parameters $h$ and $d$ appears as a trade-off: the monopole has to be buried sufficiently in the medium, and the frequency variation of $S_{11}(\theta)$ has to show marked resonances at real permittivities associated with common soils ($3 \leq \varepsilon' \leq 15$). The antenna is fed back by a coaxial transmission line with impedance $Z_a$. The measurement of the effective relative complex permittivity of a soil defined by:

$$\varepsilon_r(\theta) = \varepsilon'(\omega) - j\varepsilon''(\omega)(1 - j\tan\delta(\omega))$$  \hspace{1cm} (1)

consists in burying the monopole probe in the medium until the circular ground plane is positioned at the air-sample interface. The radiating properties of the probe in a given medium as a function of the frequency can be measured in terms of the complex reflection coefficient $S_{11}(\omega)$ ($\omega = 2\pi f$) at the input of the antenna using a VNA. The dielectric properties of the medium expressed by its relative complex permittivity $\varepsilon_r(\omega)$ are directly related to the variation of parameter $S_{11}(\omega)$ as compared to the situation where the probe is surrounded by air. The reflection coefficient $S_{11}(\omega)$ is related with the input impedance $Z_{11}(\omega)$ of the antenna by:

$$S_{11}(\omega) = \frac{Z_{11}(\omega) - Z_a}{Z_{11}(\omega) + Z_a}$$ \hspace{1cm} (2)

with $Z_a$ the characteristic impedance of the coaxial transmission line that is assumed to be equal to 50 $\Omega$.

III. ANTENNA MODELING

As the ground plane of the monopole is in practice finite, several approaches based on analytical expressions have been developed by different authors to determine the input impedance $Z_{11}$ of the antenna. The fundamental models collected by Weiner (Weiner, 2003) [12] are the following: the Richmond’s (Richmond, 1984) Method of Moments (MoM) (radius $d$ not too large compared to the wavelength $\lambda$ in the given medium), the Awadalla-Maclean’s (Awadalla, 1979) Method of Moments combined with the Geometrical Theory of Diffraction (GTD) ($d \geq \lambda$), the Storer’s variational method ($d \gg \lambda$) (Storer, 1951), and the method of images ($d \rightarrow \infty$). In general, the results show that, to be assumed infinite, the diameter $d$ of the disk should at least equal to $4\lambda$ which will usually represent a large dimension as compared to the dimension of the resonant wire element at a given resonant frequency $(2n + 1)\lambda/4$ ($n$ is a positive integer). If the disk radius can be assumed infinite, the image theory is valid, thus leading to simplified models whose validity depends on the ratio $h/\lambda$ of the monopole length relative to the wavenumber: the short monopole model (Tai and Smith) (Smith et al.,
1985), the Wu’s long antenna model (\( \hat{h} h \geq 1 \)) (Olson et al., 1986, Wu, 1961), and the induced EMF method (\( l / \lambda \leq 1 / 4 \) or \( l / \lambda \leq 1 \)) (Otto, 1969, Misra, 1988).

In the present work, we have looked for analytical models describing the frequency behaviour of \( S11(\omega) \) for a long and a short monopoles in order to solve more easily the inverse problem; the use of numerical modelling in parallel has allowed to validate the analytical modeling for the lengths defined and associated with both types of monopoles.

3.1 Modeling of the long monopole

The analytical modeling of a coaxial probe composed of an extended inner conductor immersed in a dielectric and dispersive medium is derived from the development of T.T. Wu in 1961 (Wu, 1961) [2] which was concerning a long thin-wire dipole antenna (a/\( \lambda \ll 1 \), \( a/h \ll 1 \) and \( h \gg \lambda \)).

In his modeling, T.T. Wu has expressed rigorously the input impedance of such a coaxial probe. The development is, therefore, not limited to small values of \( h/\lambda \) and has been used by S.C. Olson (Olson et al., 1986) [3] to model the behaviour of a monopole antenna with a finite circular ground plane using the image theory. In such a case, the monopole input impedance is estimated to half the dipole impedance value. In our application, the antenna geometry has been defined as follows: \( h = 6 \, \text{cm}; \ a = 0.5 \, \text{mm}; \ d = 15 \, \text{cm} \).

The complex input impedance \( Z11 \) of the ground plane monopole is expressed as follows:

\[
Z11(\omega) = \frac{\omega \mu_0}{j4k(S+CU)} \quad \text{[Ohms]} \tag{3}
\]

where in this model the several parameters involved are the following:

\( \varepsilon_{\text{rel}}(\omega) = \varepsilon' + j\varepsilon''(\omega) = \varepsilon' + j \frac{\sigma}{\omega \varepsilon_0} \tag{4} \)

is the relative complex permittivity of the dissipative medium (the positive sign is associated with this model only);

\( k = \beta + j\alpha = \omega \sqrt{\mu_0 \varepsilon_0} \left( \varepsilon' + j \frac{\sigma}{\omega \varepsilon_0} \right)^{1/2} \tag{5} \)

is the associated wavenumber in the medium.

And:

\[
\begin{align*}
C &= -\frac{1}{2} \left( 2T - T' \right) \sin(kh) - \left( 2S - S' \right) \cos(kh) \\
U &= -j \left( A_z + A_y \right) \\
S &= \frac{1}{2} \left( -A_z + A_x + A_y \right); \quad S' = \frac{1}{2} \left( -A_z + A_x - A_y \right) \\
T &= j \frac{1}{2} \left( -A_z - A_x + A_y \right); \quad T' = \frac{1}{2} \left( -A_z - A_x - A_y \right)
\end{align*}
\tag{6}
\]

with the several parameters involved in the definition of \( C, U \) and \( S \):

\[
\gamma = 0.57722 \, \gamma' = 1.6449 \tag{8}
\]

\[
\Omega_a = \ln(\lambda/a) - \ln(\gamma) \quad \Omega_0 = \Omega_a - \ln(2) \tag{9}
\]

\[
\Omega_z = 2\Omega_0 + \ln(2kh) + \gamma - j \frac{\pi}{2} \quad \Omega_{z'} = \Omega_z + \ln(2) \tag{10}
\]

\[
\Omega_1 = \Omega_z + j\pi; \quad \Omega_3 = \Omega_z + j\pi \tag{11}
\]

\[
A_1 = \ln(1 + j\pi / \Omega_0') + \frac{\pi^2}{12} \left( \frac{1}{\Omega_0' - \ln(2)} - \frac{1}{\Omega_0' - \ln(2) + j\pi^2} \right) \tag{13}
\]

\[
A_2 = \ln(\Omega_0'/\Omega_{z'}) + \frac{1}{2} \gamma (\Omega_0'^2 - \Omega_{z'}^2) \tag{13}
\]

Since the input impedance \( Z11 \) cannot be measured directly by a VNA, it is measured in terms of the reflection coefficient at the antenna feed point according to relation (3).

3.2 Modeling of the short monopole

In the approach first developed by Smith and Norgard (1985) [4], a rational polynomial function of the electrical length \( k h \) of the antenna in a given medium (of order 3 in the denominator, and assuming \( k h \ll 1 \) ) has been used to represent the normalized impedance of a short monopole, and its coefficients have been determined using a system of equations from the measurement of the input impedance in a standard medium; the algebra appears tedious, but straightforward. Afterwards, He and Shen (1992) [13], and Ballard (1993) [14] have proposed a polynomial of order 5 at the denominator which has been solved using the least-square criterion associated with a standard medium, and two antenna lengths. Such an approach will be used in the present work, but only one antenna length will be considered. In our application, the antenna geometry has been defined as follows: \( h = 6 \, \text{cm}; \ a = 0.5 \, \text{mm}; \ d = 15 \, \text{cm} \).

The measured impedance \( Z11(\omega) \) of the antenna immersed in a medium of relative complex permittivity \( \varepsilon_{\text{rel}} \) is related to the normalized impedance \( Z_s(\omega) \) as:

\[
Z11(kh) = \sqrt{\varepsilon_{\text{rel}}} \left( \frac{kh}{k_{s}h} \right) Z_s(kh) \tag{14}
\]

From the definition of the wavenumber, we then have:

\[
\varepsilon_{\text{rel}} = \left( \frac{kh}{k_{s}h} \right)^2 \tag{15}
\]
where $k$ is the wavenumber in air.

Assuming a short antenna, the normalized impedance $Z_n(kh)$ may be approximated in the following rational form:

$$Z_n(kh) = \frac{kh}{k_h} 11(kh) = \frac{K}{kh} \left[ 1 + j b_1(kh) + b_2(kh)^2 + ... + b_n(kh)^n \right]$$

(16)

where all the coefficients $a_i, b_i, a_2, ..., a_n, b_n$, and $K$ are supposed to be real.

For practical calculations over a limited range of $kh$ (for a given frequency band), the numerator and denominator of relation (16) must be estimated by polynomials of finite degree (order $m$ for the numerator, and $m + 1$ for the denominator). Because of the non linear form of relation (17), analytical methods cannot be used to determine the several coefficients. Considering a reference medium (for example air with index $s$), since both the measured data and the function are complex, we propose to minimize the sum of the distances between the data samples $Z^{'\text{num}}(k, h)$ and the theoretical points $Z^{'\text{theo}}(k, h)$ in a given frequency band $(f_1, ..., f_m, ... f_{\text{max}})$ using the least square criterion:

$$\text{Min}(S) = \text{Min} \left\{ \sum_{k=1}^{N} \left| \frac{N(k, h) x (kh)}{Z^{'\text{theo}}(k, h) x (kh)} - D(k, h) \right|^2 \right\}$$

(17)

where:

$$Z^{'\text{theo}}(k, h) = \frac{N(k, h) x (kh)}{D(k, h)}$$

(18)

An initial value for $K$ in equation (16) can be defined at low frequencies $|kh| \leq 1$ using the following relation:

$$K = \lim_{kh \to 0} \left( \frac{f(kh)^2}{kh} - Z^{'}(kh) \right)$$

(19)

3.3 FDTD modeling

Numerical simulations based on the FDTD approach have been developed under the commercial software EMPIRE. The dimensions of the structure can be visualized on Fig. 3 in the case of a cylindrical probe of length $h = 6 \ cm$ and radius $a = 0.5 \ mm$ with a circular ground plane radius of $d = 15 \ cm$, and thickness $\varepsilon = 0.01 \ mm$. The source, assumed to be punctual and directed along direction 0z, has been positioned between the bottom end of the wire element and the circular ground plane ($z = 0$ and $z = 1.56 \ mm$). The wire element is immersed in an upper semi-infinite dielectric layer ($z \geq 0$) representing the soil; its complex permittivity can be described with a frequency variation law. A semi infinite layer of air is positioned in the lower horizont-
amplitude decrease of mark that the analytical modeling does not reproduce an modeling and the analytical one that is towards the left for shift for the other resonant peaks between the numerical close (see Tables 1 and 2). But, we observe a frequency
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values, the attenuation produced is greater than 10 dB. Moreover, we remark that the presence of a given conductivity in the medium causes an attenuation of the amplitude \( S11(dB) \) versus frequency without modifying its shape when \( \sigma \) is not too high (\(< 0.1 \text{ S.m}^{-1} \) according to Fig. 7).

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Then, fixing the value of the real permittivity (\( \varepsilon' = 5 \)), the influence of the conductivity of the medium surrounding the probe has been studied. From Fig. 4, it can be remarked that a conductivity less than \( \sigma = 0.1 \text{ S.m}^{-1} \) induces a slight attenuation of the reflection coefficient. For higher conductivity values, the attenuation produced is greater than 10 dB.

In the case of a long monopole, as visualized on Fig. 6, the comparison of the results issued from the analytical and the numerical (FDTD) models for real permittivity values ranging from 1 to 7 (\( \sigma = 0 \)) highlights that the positions of the first resonant peaks calculated by both models appear very close (see Tables 1 and 2). But, we observe a frequency shift for the other resonant peaks between the numerical modeling and the analytical one that is towards the left for \( \varepsilon' < 5 \), and towards the right for \( \varepsilon' \geq 5 \). Moreover, we remark that the analytical modeling does not reproduce an amplitude decrease of \( S11(dB) \) associated with the multiple reflections observed along the frequency band considered. Then, studying the influence of the imaginary permittivity by means of the conductivity \( \sigma (\varepsilon'' = 5) \), we remark from Fig. 7 that a frequency shift of the resonant peaks as compared to the case where \( \sigma = 0 \) is observed when the conductivity reaches a value higher than 0.05 S.m\(^{-1} \). In general, we observe that the first step consists in estimating the real and imaginary parts of the complex permittivity of a soil in a given frequency band using the monopole probe with characteristics, we propose several algorithms that are detailed below.

**Table 1: Resonant frequencies associated with the numerical FDTD model (\( \sigma = 0 \))**

<table>
<thead>
<tr>
<th>Soil permittivity</th>
<th>( f_1 ) [GHz]</th>
<th>( f_2 ) [GHz]</th>
<th>( f_3 ) [GHz]</th>
<th>( f_4 ) [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon' = 1 )</td>
<td>1.12</td>
<td>3.44</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon' = 2 )</td>
<td>0.83</td>
<td>2.48</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon' = 3 )</td>
<td>0.69</td>
<td>2.07</td>
<td>3.44</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon' = 4 )</td>
<td>0.61</td>
<td>1.83</td>
<td>3.03</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon' = 5 )</td>
<td>0.56</td>
<td>1.67</td>
<td>2.75</td>
<td>3.84</td>
</tr>
<tr>
<td>( \varepsilon' = 7 )</td>
<td>0.48</td>
<td>1.45</td>
<td>2.38</td>
<td>3.30</td>
</tr>
</tbody>
</table>

**Table 2: Resonant frequencies associated with the analytical long antenna model (\( \sigma = 0 \))**

<table>
<thead>
<tr>
<th>Soil permittivity</th>
<th>( f_1 ) [GHz]</th>
<th>( f_2 ) [GHz]</th>
<th>( f_3 ) [GHz]</th>
<th>( f_4 ) [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon' = 1 )</td>
<td>1.18</td>
<td>3.63</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
5.1 Case of the short monopole

Previous studies have highlighted that if the modeling of the normalized impedance $Z_n(kh)$ with a rational polynomial in a given frequency band has to show one resonant frequency, the minimal degree of the function must be $m = 4$. Thus, impedance $Z_n(kh)$ can be rewritten as follows:

$$Z_n(kh) = \left( \frac{kh}{c} \right)^m \sum_{i=0}^{m} c_i(kh)^i$$

(20)

With $c_i = K$.

The algorithm developed for the determination of the complex permittivity of a soil involves the following steps:

1) Determination of the 9 coefficients of the rational polynomial function (the calibration process) which are mainly associated with the antenna geometry; thus, a reference medium (index $s$) such as air is chosen, and the minimal distance between the measured data samples $Z_{\text{data}}(k, h)$ and the theoretical points $Z_{11}(k, h)$ for each frequency samples (index $t$) in the frequency band has to be found according to the least square criterion:

$$\text{Min}(S_t(f = f_t)) = \text{Min}\left[ \frac{\sum_{i=1}^{N} (k, h) x (k, h) - Z_{\text{data}}(k, h) x (k, h)}{D(k, h)} \right]$$

(21)

The measured impedance $Z_{\text{data}}(k, h)$ is obtained from the antenna reflection coefficient $S_{11\text{data}}(k, h)$ using relation (2).

2) Extracting the measured data of $S_{11\text{data}}(k, h)$ and smoothing them if necessary using a low-pass filter with a finite impulse response of order $N$. The filtering consists in replacing the complex sample $S_{11}(\omega_s)$ at a given frequency $f_s$ by the average complex value of the $N$ nearest samples.

3) Determining the roots of the polynomial of degree 6 (6 solutions) for each frequency belonging to the bandwidth of the calibration made in air; the bandwidth may be reduced, particularly the higher frequencies in the case of the visualization of more than one resonant frequencies; in such a case the rational polynomial function will represent correctly the shift towards the left (because of the weaker propagation speed in a dielectric soil compared to that of air) of the resonant peak. Thus, to represent more than one resonant peak, the rational polynomial function must be of higher degree.

Defining $X_i = kh$:

$$Z_{11}(X_i) = (kh) \frac{jc_i - c_s(X_i)}{jc_s(X_i)}$$

$$Z_{\text{data}}(X_i) = (kh) \frac{jc_i - c_s(X_i)}{jc_s(X_i)}$$

(22)

the objective is then to determine the roots of the following polynomial of degree 6:

$$Z_{11}(X_i) - Z_{\text{data}}(X_i) = 0$$

(23)

$$\Rightarrow (kh) [jc_i - c_s(X_i)] = Z_{\text{data}}(X_i) [jc_s(X_i)]$$

$$\Rightarrow (kh) [jc_i - c_s(X_i)] = Z_{\text{data}}(X_s) [jc_s(X_i)]$$

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$$\Rightarrow (kh) [jc_i - c_s(X_i)] = Z_{\text{data}}(X_s) [jc_s(X_i)]$$

(24)

4) Sorting the 6 possible solutions of the complex permittivity at each frequency; the final solution has to follow several constraints such as: the real part must be always positive, and the imaginary part has to be negative. Moreover, the variations of the real and imaginary permittivities versus frequency must not show discontinuity, so at a given fre-
quency the values closest to the values estimated at the previous frequency are retained.

5.2 Case of the long monopole
To estimate the complex permittivity of a medium in a given frequency band we have developed 2 algorithms.

Algorithm based on the resonant frequencies
The steps used are the following:
- Detection and selection of local minima (resonant frequencies) of a given experimental curve $S11(f)$ associated with a value of real permittivity value $\varepsilon'' = 0$ by the determination of the gradient, and low-pass interpolation of the 17 nearest frequency samples around each minimum frequency;
- Extension of the minima detection of $S11(f)$ to a range of real permittivities for data issued from both analytical and numerical modeling;
- Plotting the variation of the first three resonant frequencies as a function of the real permittivity for both modeling, and determination of the parameters $(a_i, b_i)$ of the curve $f_i(\varepsilon')$ fitting the variation of each resonant frequency versus the real permittivity; each curve $f_i(\varepsilon')$ represents a square root variation versus $\varepsilon'$ such as:

$$f_i(\varepsilon') = a_i + \frac{b_i}{\sqrt{\varepsilon'}} \hspace{1cm} i = 1,2,3$$  \hspace{1cm} (24)

- Determination of the average variation of each resonant frequency versus the real permittivity according to both curve fitting associated with both modelling;
- Estimation of the real permittivity from the first three resonant frequencies issued from the experimental data of $S11(f)$ and compared to the mean fitting curves, using the least square criterion.

Algorithm based on the high resolution Prony approach
We propose an original approach which aims at the identification of the individual paths issued from the successive reflections occurring inside the probe, using a parametric High-Resolution (HR) spectral analysis algorithm based on the Prony method (Sarkar et al., 1995, Qiu et al., 1999) [15]. Thus, the reflection time response of the probe surrounded by a dielectric medium is modelled as the sum of M individual paths which represent multiple copies of the incident signal, the k-th path being characterized by a complex delay $\tau_k$ and a complex attenuation $a_k$. Considering M paths, the transfer function writes as follows:

$$H(\omega) = \sum_{k=1}^{M} a_k e^{-j\tau_k \omega} \hspace{1cm} \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}}$$  \hspace{1cm} (25)

where $\omega = 2\pi f$.

In the present modeling, delay $\tau_k$ is a complex number and is defined as follows:

$$\tau_k = T_k - j\alpha_k$$  \hspace{1cm} (26)

where $T_k$ represents the travel time of path k through the medium, and $\alpha_k$ is the slope of the frequency attenuation. Such a complex delay allows to take into account the frequency dispersion involved in wideband studies. Relation 26 can be rewritten such as:

$$H(\omega) = \sum_{k=1}^{M} a_k e^{-j\tau_k \omega} = \sum_{k=1}^{M} b_k Z_k = \sum_{k=1}^{M} H_k(\omega)$$  \hspace{1cm} (27)

where $b_k = a_k e^{-j\tau_k}$, and $Z = e^{j\alpha \omega}$, $\omega = \omega_0 + n\omega_1$ is the angular frequency of the $n$-th sampled spectral component ($1 \leq n \leq N$).

The data processing consists in fitting the frequency data (at present, the reflection coefficient $S11$) to the model (expressed by relation (3)) to estimate the complex amplitudes and exponential arguments of the individual delayed paths k. Afterwards, assuming the Hermitian symmetry, the impulse response associated with each detected path is analytically determined. Associated with a parametric modeling, the Pencil Matrix (PM) and Eigen-Pencil Matrix (EPM) algorithms, proposed by Sarkar and Qiu (Sarkar et al., 1995, Qiu et al., 1999) [15], aim the determination of the amplitudes $a_k$ and the exponential parameters $\tau_k$ in two main steps. At first, the exponential argument of each ray is calculated from a Singular Value Decomposition (SVD) of a specific data matrix, followed by the formation of a matrix pencil whose eigenvalues are the M poles $Z_k$ - the data matrix have specific definition in both PM and EPM algorithms. Then, the amplitudes $a_k$ are determined using a least-square fitting of the N frequency data. Once the parameters $a_k$ and $\tau_k$ associated with the M principal paths have been determined, the evaluation of the impulse response associated with these paths can be determined using a Fourier transform; thus, we assume that each spectral response has the Hermitian symmetry property as follows:

$$H_k(\omega) = \begin{cases} a_k e^{i\tau_k} e^{j\omega \tau_k} & \omega > 0 \\ a_k e^{-i\tau_k} e^{-j\omega \tau_k} & \omega < 0 \end{cases}$$  \hspace{1cm} (28)

The average dielectric properties of the surrounding medium can be estimated from the complex propagation times $\tau_k$:

$$\begin{cases} \tau' = \left(\frac{c}{2(k-1)}\right)^2 ((T_k - T_1)^2 - a_k^2) \\ \tau'' = \left(\frac{c}{2(k-1)}\right)^2 (2(T_k - T_1) a_k) \end{cases} \hspace{1cm} k = 2,...,M$$  \hspace{1cm} (29)

If a frequency dependent imaginary permittivity is assumed such as $\varepsilon'' = \frac{\sigma}{\omega \varepsilon_0}$, then the conductivity value $\sigma_k$ can be
estimated from the average value of $\varepsilon''$ at the maximum frequency of the band considered, leading to:

$$\sigma_{\text{rms}} = \omega_{\text{max}} \varepsilon_0 \varepsilon''_{\text{rms}}$$  \hspace{1cm} (30)$$

Afterwards, the second step consists in expressing the dependence of the real permittivity with the volumetric water content $\theta$. Thus, two forms have been considered: a polynomial fit of degree 3 similar to the empirical Topp et al. relation (Topp et al., 1989) [5], and the Complex Refractive Index (CRIM) method [7]. Considering the first form, the objective was to determine for each type of soil the 4 coefficients of the following polynomial using the least mean square criterion:

$$\varepsilon_{\text{eff}} = a_1 + a_2 \theta + a_3 \theta^2 + a_4 \theta^3$$  \hspace{1cm} (31)$$

and to compare them to the ones issued from the Topp’s relation where: $a_1 = 3.03$, $a_2 = 9.3$, $a_3 = 146$, and $a_4 = -76.7$.

Afterwards, we have proposed to use a second analytical form based on the CRIM model which has been generalized by Fratticcioli et al. (2003) [11]. It leads to the following relation:

$$\varepsilon_{\text{eff}} = K[V_s (\varepsilon_s) + V_w (\varepsilon_w) + V_h (\varepsilon_h)] + C$$  \hspace{1cm} (32)$$

This model offers the advantage of explicitly incorporating the volumetric fractions ($V_s$, $V_w$, and $V_h$) and the relative real permittivities ($\varepsilon_s$, $\varepsilon_w$, and $\varepsilon_h$) of the several constituents of the medium (mineral solids, water, and air with indices $s$, $w$, and $h$, respectively). For the soil characterized, the presence of free water has been only considered, and we have assumed that $\varepsilon_w = 81$, $\varepsilon_s = 1$, and $V_s = \theta$. The volumetric content of solids in a dry material is defined as follows $V_s = \frac{\rho_s}{\rho_d}$, with $\rho_d$ the bulk density of the dry material, and $\rho_s$ the specific density of the soil solids ($V_s + V_w + V_h = 1$). In the case of pure sand, we have assumed that $\rho_s = 2.66$ kg.dm$^{-3}$, and $\varepsilon_s = 2.56$ (the permittivity of the dry sand). With these hypotheses, relation (32) rewrites as follows:

$$\varepsilon_{\text{eff}} = K [\frac{\rho_s}{\rho_d} (\varepsilon_s) + \theta (\varepsilon_w) + (1 - \theta - \frac{\rho_s}{\rho_d})] + C$$  \hspace{1cm} (33)$$

Thus, the fitting of the measurement data with the model expressed by relation (33) supposes to determine parameters ($K$, $\alpha$, and $C$) using the least-square criterion.

It must be noted that the relations between the volumetric content of water $\theta$ and the densities $\rho_s$ and $\rho_d$ of the dry and wet soil respectively are:

$$\begin{cases}
\theta = \frac{\rho_s}{\rho_d} w \\
\theta = \frac{\rho_s}{\rho_d} w \left(1 + \frac{\rho_s}{\rho_d}\right)
\end{cases}$$  \hspace{1cm} (34)$$

where $w$ represents the moisture content by weight, and $\rho_d$ is the specific density of water.

Thus, we have easily deduced that:

$$\rho_s = \rho_d - \theta$$  \hspace{1cm} (35)$$

VI. RESULTS

The validation of the experimental tool with its inversion algorithms have been made on pure and clay sands which dielectric properties have been studied as a function of moisture content by weight ranging from 0% to 12.25%. The long monopole of 6 cm has been used here.

Pure sand has been first considered as its dielectric properties can be homogenized and it has been largely characterized by several authors [11]. The sand considered here is issued from the sandpit Stref (Jumieges, France) and belongs to class D1 (AFNOR NF P 11-300). The container in which measurements have been made, as visualized in Fig. 8 is a rectangular box composed of PVC (length 56.8 cm, wide 36.4 cm, height 40.8 cm) filled with a height of 20 cm of soil.

![Vector Network Analyzer (VNA)](image)

**Figure 8. The experimental set-up in our laboratory**

At first, the sand has been dried using a furnace heated at 107°C for 24 hours. Then, water has been added gradually to reach the different moisture contents. Compaction at a particular moisture content has been made, thus leading to the densities by weight $\rho_s$ collected in Table 3. For each moisture content, four measurement locations using the monopole have been chosen, and we have deduced average effective complex permittivities $\varepsilon_{\text{eff}}$ and conductivities $\sigma_{\text{rms}}$ using the two types of algorithms described previously. At the same time, measurements of sand samples inside a cylindrical cavity (see Tables 3 and 4) have been made (Li et al., 1981). As visualized in Fig. 9, the Prony algorithm fits very satisfactorily complex $S_{11}$ measurements associated with a medium without bound water such as sand; it must
be remarked that as the real moisture content increases the resonant frequencies are shifted towards the left.

Table 3: Dielectric characteristics of the pure sand of class D1 measured with a cylindrical cavity

<table>
<thead>
<tr>
<th>Moisture by weight</th>
<th>Cavity meas. at f₀=1.3GHz</th>
<th>Density by weight</th>
<th>ε'</th>
<th>Δε</th>
<th>ρ [kg.dm⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>2.56</td>
<td>+/-0.144</td>
<td>2.55</td>
<td>+/-0.10</td>
<td>2.09 10⁻³</td>
</tr>
<tr>
<td>2.65%</td>
<td>2.84</td>
<td>+/-0.10</td>
<td>3.12</td>
<td>+/-0.10</td>
<td>4.17 10⁻³</td>
</tr>
<tr>
<td>5.3%</td>
<td>4.11</td>
<td>+/-0.144</td>
<td>4.25</td>
<td>+/-0.08</td>
<td>5.38 10⁻³</td>
</tr>
<tr>
<td>7.4%</td>
<td>4.82</td>
<td>+/-0.14</td>
<td>5.41</td>
<td>+/-0.14</td>
<td>7.94 10⁻³</td>
</tr>
<tr>
<td>10%</td>
<td>6.09</td>
<td>+/-0.14</td>
<td>6.08</td>
<td>+/-0.1</td>
<td>1.25 10⁻²</td>
</tr>
<tr>
<td>12.25%</td>
<td>7.65</td>
<td>+/-0.14</td>
<td>7.35</td>
<td>+/-0.1</td>
<td>1.4 10⁻⁴</td>
</tr>
</tbody>
</table>

Table 6 that for the clay sand has a conductivity which increases gradually with moisture content.

Figure 9. Modulus of S11 issued from measurements of sand with several moisture contents by weight and analyzed by HR algorithm using 3 paths

Table 4: Dielectric characteristics of the pure sand of class D1 measured with the 6 cm monopole

<table>
<thead>
<tr>
<th>Moisture by weight</th>
<th>Res. Freq. ε' eff</th>
<th>Res. Freq. Δε</th>
<th>Prony algo. ε' eff</th>
<th>Prony algo. Δε</th>
<th>Prony algo. σ [S/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>2.55</td>
<td>+/-0.144</td>
<td>2.55</td>
<td>+/-0.10</td>
<td>2.09 10⁻³</td>
</tr>
<tr>
<td>2.65%</td>
<td>2.84</td>
<td>+/-0.10</td>
<td>3.12</td>
<td>+/-0.10</td>
<td>4.17 10⁻³</td>
</tr>
<tr>
<td>5.3%</td>
<td>4.11</td>
<td>+/-0.144</td>
<td>4.25</td>
<td>+/-0.08</td>
<td>5.38 10⁻³</td>
</tr>
<tr>
<td>7.4%</td>
<td>4.82</td>
<td>+/-0.14</td>
<td>5.41</td>
<td>+/-0.14</td>
<td>7.94 10⁻³</td>
</tr>
<tr>
<td>10%</td>
<td>6.09</td>
<td>+/-0.14</td>
<td>6.08</td>
<td>+/-0.1</td>
<td>1.25 10⁻²</td>
</tr>
<tr>
<td>12.25%</td>
<td>7.65</td>
<td>+/-0.14</td>
<td>7.35</td>
<td>+/-0.1</td>
<td>1.4 10⁻⁴</td>
</tr>
</tbody>
</table>

A clay sand of class B2 (AFNOR NF P 11-300) issued from the sandpit Stref (Jumieges, France) is now considered; it includes a weak amount of clay. Several concentrations of water by weight have been analyzed 2.59%, 5.07%, 7.48%, 9.26%, and 11.2%. As previously, the permittivity estimations issued from the several algorithms are collected in Tables 5 and 6.

For both sands, the relation between the real permittivity and the volumetric moisture content has been expressed using the Topp’s polynomial and the CRIM relations. As compared to pure sand, Fig. 10 highlights that the clay sand shows a marked sensibility to the presence of water, particularly at low volumetric moisture content (less than 5%); this observation is consistent with its belonging to class B2. Moreover, as can be visualized in Fig. 18, the variation of the real permittivity with volumetric moisture content of this clay sand agrees quite well with Topp’s relation. The coefficients associated with the polynomial fitting of degree 3 of the real permittivity estimates are: \( a_3 = 2.852 \), \( a_2 = 9.26 \), \( a_1 = 149.077 \), and \( a_0 = -22.907 \). In the case of the CRIM fit with \( K = 1 \), the coefficient values are: \( a = 1.901 \), and \( C = 5.29710^{-1} \). Moreover, we remark from

Table 5: Dielectric characteristics of the pure sand of class B21 measured with a cylindrical cavity

<table>
<thead>
<tr>
<th>Moisture by weight</th>
<th>Cavity meas. at f₀=1.3GHz</th>
<th>Density by weight</th>
<th>ε'</th>
<th>Δε</th>
<th>ρ [kg.dm⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>2.88</td>
<td>1.34</td>
<td>2.55</td>
<td>+/-0.10</td>
<td>2.09 10⁻³</td>
</tr>
<tr>
<td>2.59%</td>
<td>3.89</td>
<td>1.54</td>
<td>4.17 10⁻³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.07%</td>
<td>4.04</td>
<td>1.56</td>
<td>5.38 10⁻³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.48 %</td>
<td>6.39</td>
<td>1.69</td>
<td>7.94 10⁻³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.26%</td>
<td>7.87</td>
<td>1.79</td>
<td>1.25 10⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.20%</td>
<td>9.84</td>
<td>1.81</td>
<td>1.4 10⁻⁴</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Measurements have also been made with the 6 cm long monopole on a test site at LRPC Rouen where a silt of class A1 has been deposited in a chamber of 25 m long and of 1 m height with 3 compaction levels along the chamber length (weak, medium, and high); the soil density measured by a gamma probe is 1.884, 2.044, and 2.11 g/cm³ respectively. As visualized on Fig. 11, the real permittivity has been estimated using the 6 cm monopole and a commercial TDR probe with 2 fingers.

VII. CONCLUSION

This paper presents a novel tool to characterize the moisture content of soft soils using a monopole probe in a large frequency range. The validation of the experimental tool with its inversion algorithms have been made on pure and clay sands and a test site with silt. Future trends aim the study of different mixtures of soils and the use of a cross-hole radar.

Table 6: Dielectric characteristics of the pure sand of class B21 measured with the 6 cm monopole
Moisture by weight | Res. Freq. $\varepsilon'_\text{eff}$ | Res. Freq. $\Delta\varepsilon'$ | Prony algo. $\varepsilon'_\text{eff}$ | Prony algo. $\Delta\varepsilon'$ | Prony algo. $\sigma$ [S/m]
--- | --- | --- | --- | --- | ---
Dry sand | 2.13 | +/-0.14 | 2.32 | +/-0.10 | 1.89 $10^3$
2.59% | 3.40 | +/-0.14 | 3.49 | +/-0.10 | 3.8 $10^3$
5.07% | 4.25 | +/-0.14 | 4.40 | +/-0.10 | 6.24 $10^3$
7.48% | 6.37 | +/-0.28 | 6.47 | +/-0.20 | 9.28 $10^3$
9.26% | 7.22 | +/-0.56 | 7.01 | +/-0.20 | 1.36 $10^2$
11.20% | 9.48 | +/-0.35 | 9.43 | +/-0.20 | 1.86 $10^2$

Figure 10. Comparison of the variation of the real permittivity versus volumetric moisture content for pure and clay sand analyzed by the polynomial and the CRIM approaches.

Figure 11. Comparison of the real permittivity estimate on the test site with silt using the 6 cm monopole and a TDR probe.

REFERENCES